PURPOSE-LED
PUBLISHING™

PAPER • OPEN ACCESS

A review on straightening of bars and application of probabilistic approach on Moment-curvature relationship

To cite this article: Sanjib Roy *et al* 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **810** 012080

View the article online for updates and enhancements.

A review on straightening of bars and application of probabilistic approach on Moment-curvature relationship

Sanjib Roy[1]
Narsee Monjee Institute of Management Studies, India
sanjibbhel@gmail.com

Arun Kiran Pal[2]
Jadavpur University, India
arunkiranpal@yahoo.co.in

Shannay Rawal[3]
Narsee Monjee Institute of Management Studies, India
shannay.nmims@gmail.com

Abstract. Bar straightening is a significant process in the value stream of bar manufacturing processes. Any improvement in the process has a substantial impact on quality and productivity as a whole. This paper presents the outline of the fundamentals of the straightening process from a material science perspective and discusses the state of art in terms of process analysis and development. It has been inferred that the major aspect of the process is to understand the moment-curvature relationship in bars subjected to re-rolling. The process is governed by several parameters such as bar speed, the angle between osculating plane and loading plane, bar size or radius of bar, applied reverse bending moment, initial and final curvature, etc. It has also been observed that the research in this domain has seldom applied a statistical approach to enhance process capability. A brief account of the potential use of statistical process control has been presented. Use of statistics shall enable a better understanding of various process parameters. This, in turn, will help in deciding whether the process is in control and therein to assist in deciding the acceptance or rejection of lot.

Nomenclature:

c = a constant
E = Elastic modulus
E_p = Plastic modulus of the bar.
σ = Stress
ϵ, e = strain
Y = Yield Point, Yield Stress
M = Bending moment for producing change in curvature, κ
M_o = Moment needed to produce zero curvature
κ = curvature change along length of bar.
κ_r = residual curvature along bar length
κ_f = final residual curvature along bar length
η = curvature function
$\sigma_a, \sigma_b, \sigma_c$ = stresses at point a, b and c
h = depth of the section of bar
ϵ_y = yield strain

ϵ_r = residual strain

ϵ_{r1} = residual strain at extreme fiber

ϵ_1 = change in strain at ½ depth of bar section or at *h/2* in fiber

λ = a function dependent on section parameter, line segment

I_z = Moment of inertia about z-axis

v_x = Throughput speed of bar

θ = Angle between osculating plane and loading plane

r = radius of bar

P = Probability of occurrence

$f_\kappa(\kappa_f)$ = probability density function of final curvature

μ_κ = mean curvature

σ_κ = standard deviation of curvature

1. Introduction:

Bar Straightening is a process of straightening of bars of various sections in order to improve degree of straightness [1]. Straight Bars of various sections are a general requirement in industries. Straight Bars are required as raw material for the purpose of production of various parts, machine components etc. in industries. There are many large number of parts that are formed from cut pieces of straight bars. Hence, degree of straightness [1] is always a general concern in any industry where straight bars are required as raw materials where straightness of rollers, rolling shafts, idlers are significant machine components. In printing machineries, rolling shafts, idlers are used in significant numbers from start of feed to end products. In all such cases use of straight bars have significant importance for smooth operation of machines and quality output.

In re-rolling industries degree of straightness is of prime consideration. Rolling mills provide light sections and bars in slightly bent and distorted conditions, which need to be straightened before end use. Commercial straightness of metallic bars and sections usually ranges from 1 in 750 to 1 in 5000 depending on their application.[1]

Cross-roll straightening machines are widely used to straighten circular bars and tubes. The product axis is used as a reference about which the bar is rotated as it moves along the axis through a series of staggered rolls.[2]   The principle of operation is used as a primary classifier for commercially straightening machines. The main groups are section straighteners, cross-roll straighteners and stretch straighteners.

The product bar passes through a series of staggered rolls in section straighteners and cross-roll straighteners. Plastic strains are redistributed in the bars due to the presence of reverse kinematic loading. In the process it secures the desired straightening [2]. Several work have been done on bar straightening, mostly in the theoretical development with general analysis of straightening process[2] including numerical modelling approach, experimental simulation[22], optimization, etc., of straightening process.

In the re-rolling industries, statistical considerations are of much importance since many quality related decisions are usually based on statistical considerations. Rejections of products are also due to process based on control limits or specification limits set by quality standards. It is always intended to keep the process close to estimated value or mean value of the desired parameters. Rejection of products if takes place do necessarily have an impact on overall cost of production and ultimately on the price of final product.

In the subsequent sections, the background of the straightening process is reviewed. In addition, an account of introducing statistical consideration on controlling parameters in the straightening process is presented.

2. Mechanics of Bar Straightening

Bar straightening involves applications of stresses by the rolls during the axial travel of the bars. To understand the mechanism, an analysis was carried out by "Das Talukder, N.K. et al." This analysis was based on several assumptions as listed below.

  i.   Elastic or linear work hardening is assumed to be present in the material of the straightened bar. [12] [Figure 1a]

  ii.   Under the condition of pure bending, the bar has a circular cross section and the neutral axis passes through the centre of the bar end axes. [12]

  iii.   The cross section remains on a plane and always make right angles to the axis of the material.[9]

  iv.   At any assumed point in the bar section, the strain varies proportionally with the distance from the perpendicular point on the neutral axis. [12]

  v.   The material is perfectly elastic-plastic, and its stress-strain relations in uniaxial tension and compression are the same. It has the same modulus of elasticity and yield point in tension and compression.[9]

  vi.   Every stress component other than the stress in longitudinal direction is zero.[9]

The basics of stress-strain relationship have been stated for enabling understanding how the analysis has actually proceeded. We are familiar the stress-strain relationship of any isotropic material having elastic modulus, E up to elastic limit and plastic modulus, E_p beyond E. A simplified diagram on the same is presented below [Figure 1a]. Straightening of bar essentially involves moment-curvature relationship as reverse bending due to applied moment results in reduction of curvature. This is quite a usual practice in industry and also experimentally shown by [Biju, B et.al. IRJET, 2016] [23].
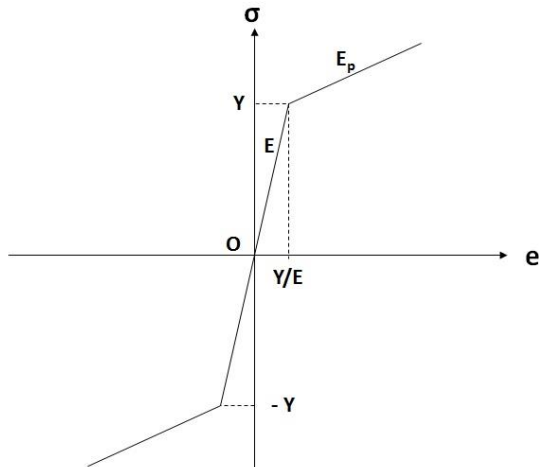

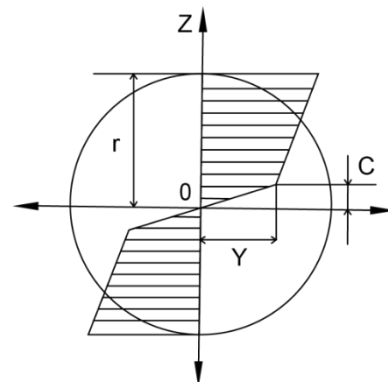**Figure 1a**: The Stress-strain relationship of the material

Figure 1b: The Stress distribution through the section of a circular bar when subjected to pure elastic-plastic bending

where E = Elastic modulus and E_p is plastic modulus of the bar.

This phenomenon is well explained by Bauschinger effect.

The distribution of stresses in the bar cross section when subjected to elastic-plastic pure bending which can be observed in Figure 1b. The stress-strain relationship is customarily expressed as below:

$$\sigma = E\varepsilon \qquad\qquad\qquad for \qquad\qquad z < c$$
$$= Y + E_p \left[\varepsilon - \frac{Y}{E}\right] = (1-\mu)\, Y + E_p\, \varepsilon \qquad for \qquad\qquad z \geq c$$

where, $\mu = \dfrac{Ep}{E}$, E is elastic modulus and E_p is plastic modulus and Y is yield point.

2.1 Bauschinger effect [18]

Previous research findings have found that repeated plastic deformation introduces three major effects in pipes and bars: strain hardening, Bauschinger effect and strain aging. However, point of interest is much restricted to mainly on Bauschinger effect. This can be attributed to the cyclic loading and unloading of bars or pipes in reverse directions in addition to the unidirectional loading beyond σ_b when the initial yield point is at σ_a. [Figure 1c].

Bauschinger effect is a metallurgical phenomenon wherein the stress/strain characteristics of the studied material change due to the influence of microscopic stress distribution in the material.[18] The effect is notably observed in real metals, whenever there is a reversal of stress. In the process of re-rolling, consequently, there is a change in yield point in reverse loading.
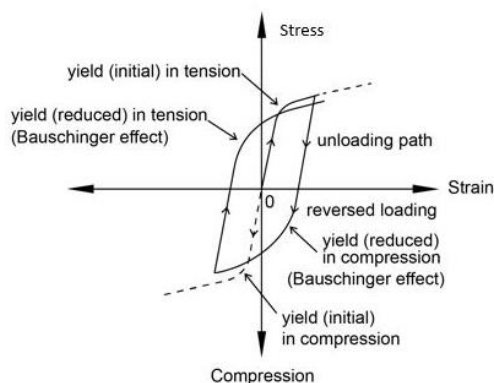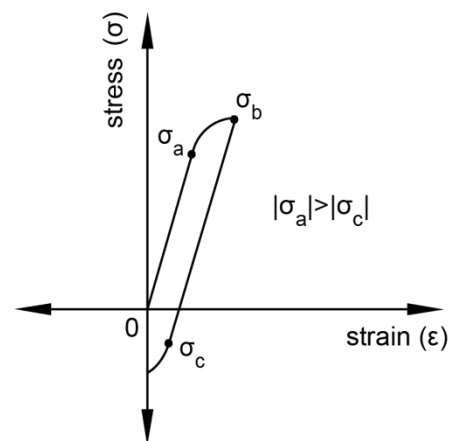

Figure 1c   **Figure 1d**

Figure 1c & 1d: Bauschinger effect

The above-mentioned loadings are widely regarded as the cause for occurrence of Bauschinger effect. This is due to the close association of the result where after initial plastic deformation, reverse yield strength σ_c is reduced in comparison to the initial value [Figure 1d].

There are three different known residual stresses, viz. Type-1, Type-2 and Type-3 residual stresses. Commercially manufactured components possess all three residual stresses in varying degrees that are generated during the process of manufacturing. Type-2 and Type-3 residual stresses are primary causal agents of Bauschinger effect.[3][4]
The physical presence of these stresses could be imperceptible or in the form of minute cracks. Cast and welded components can fail catastrophically even without loading at room temperature due to residual stresses. They are a proven agent in reducing the working lifespan of metal components.[5]

As further discussion is mainly on reverse loading of bars, our consideration will be made on such analysis where discussion is centred around reverse loading and its impact on final curvature, consequently to degree of straightness and statistical consideration on the same.

3. Analysis of Bar Straightening

Detailed analysis of the involved mechanics has been mainly done by Das Talukder, Johnson and Yu in the processes of straightening of bars.[2][6][12] In the analysis, it has been assumed that bars behave both anisotropically as well as non-homogeneously, along with the degree of straightness of the finished component.[6] More thorough analyses of the processes have also been done for different types of machines.[2]

When analysing the process of general straightening under the action of reverse kinematic loading, prime considerations are of curvature change and resisting moments. Assumption is made that stress-strain curve on the basis of the findings of Davis et. al. are indicative curves showcasing the results for ductile nature of materials.[4] The stress-strain relationship, between the reverse loading stress, σ and the change in longitudinal strain, ε [Figure 2(a)] is collectively demonstrated in Figure 2(b).
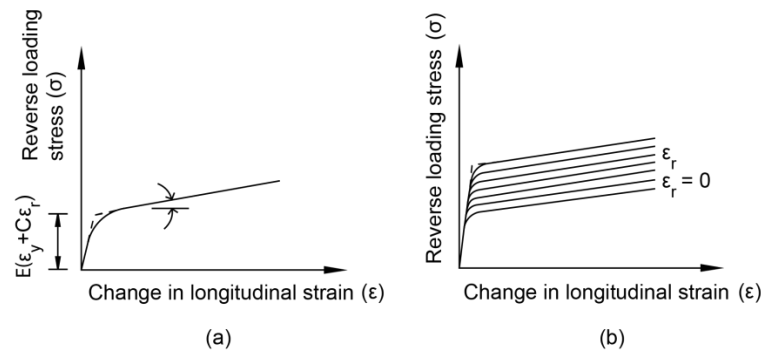

Figure 2. Stress-strain change curves for various different initial residual strains.

Based on the results as reported[6][7], it shows that reverse loading does not have a clearly defined yield point and instead curves show sharp changes in the yield region. In addition to assumptions made earlier, it is also necessary that following assumptions are made:

(a) Yield point stress is the same in tension and in compression for an initially unstrained bar.

(b) During reverse loading, stress-strain relation is bilineal. This relationship is very close to the results reported by Davis, E. A, et al (1950) [7]. The bilineal relationship as assumed can be expressed as

$$\sigma = E\varepsilon \qquad\qquad\qquad\qquad\qquad\qquad \varepsilon < (\varepsilon_y + c\varepsilon_r)$$
$$\sigma = E(\varepsilon_y + c\varepsilon_r) + E_p[\varepsilon - (\varepsilon_y + c\varepsilon_r)] \qquad \varepsilon \geq (\varepsilon_y + c\varepsilon_r) \qquad\qquad (1)$$

Where, E is elastic modulus and E_p is plastic modulus,

ε_y is the yield strain during reverse loading without initial residual strain for an element,

c is a constant   and $c \ll 1$.[18]

3.1 The Moment ~ Curvature (M ~ κ) relationship for Circular Bars

Considering a bar with the following characteristics:

  i.   Sectional symmetry about two axes in the y and z directions such that they are mutual perpendicular.

 ii.   initial residual curvature κ_r in one plane of symmetry which will be different at different distances since strains in fibre at different distances will be different from z-axis.

Figure 2(b) shows the variation in strain change-stress curves as a result of an induced strain change. According to the fibre in consideration, the curve of the initial residual strain is noted to calculate the stress.

If residual strain in the extreme fibre is ε_{r1} and the residual strain in the fibre at a distance y from the z-axis is ε_r then assuming h to be the depth of the studied section, ε_r to be the residual strain at a distance of y units from the z-axis and the residual strain in the furthest fibre to be ε_{r1} then

$$\frac{\varepsilon_{r1}}{h/2} = \frac{\varepsilon_r}{y} = \kappa_r \tag{2}$$

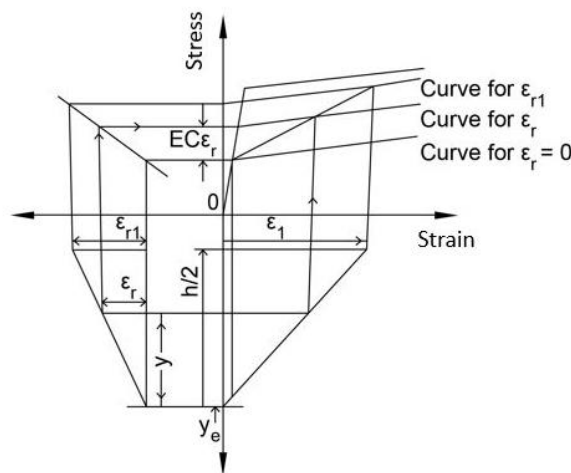If κ is the change in curvature along the length of the bar as a result of the straightening, then from Figure 3:

Figure 3:   Variation of stress across the depth in bending

$$\kappa = \frac{2\varepsilon_1}{h}$$
$$\text{and } \varepsilon = \frac{2y\varepsilon_1}{h} \tag{3}$$

where the change in strain in the fiber at distances y and $h/2$ are given by ε and ε_1 respectively. If the value of y at elastic-plastic boundary is given by y_e,[6] then from equation (3)

$$
\begin{aligned}
\sigma &= \kappa E y & -y_e \leq y \leq y_e \\
&= (E - E_p)\,\varepsilon_y + \xi_y & -y_e \leq |y| \leq \frac{h}{2} \\
&\text{where } \xi = (E - E_p)\,c\kappa_r + E_p\kappa
\end{aligned} \tag{4}
$$

Thus, the moment required to produce this change, M, can be found as follows [6]:

$$
\begin{aligned}
M &= [\lambda E\kappa + (1-\lambda)\,\xi]\,I_z & \kappa \geq (\kappa_y + c\kappa_r) \\
& & 0 \leq \lambda \leq 1 \\
&= \kappa E I_z & \kappa \leq (\kappa_y + c\kappa_r)
\end{aligned} \tag{5}
$$

where λ is a function dependent on the section parameters, y_e and change in curvature.

We know, $\qquad\qquad\qquad y_e = \dfrac{\varepsilon_y}{h}$ $\qquad\qquad\qquad\qquad\qquad$ (6)

If the moment required to produce the same curvature change κ in a bar length, having zero initial curvature is represented by $M_o(\kappa)$, then from equations (4) and (5),[6]

$$M_o = [E + (1 - \lambda)\, E_p]\, \kappa I_z \qquad\qquad\qquad (7)$$

from equations (5) and (7)

$$\Delta M = M - M_o = (1 - \lambda)(E - E_p\,]\, c\kappa_r\, I_z \qquad\qquad (8)$$

where M is the extra moment required to produce a particular curvature, due to initial curvature in the bar. It has been seen that the curvature falls down from roller to roller (Groover, 2010)[24] when a round bar is subjected to reverse bending according to the absolute values.[23]

4. Statistical approach to the Bar Straightening

It has been seen experimentally that deflection reduces after initially bent bar passes through straightening process by reverse bending. Three specimens have been used by Biju et. al. and experimentally observed that at given points deflections reduced in each specimen.[23] Considering the data published by Biju et. al. for three different specimens, it has been observed that changes in deviation from central axis are much dominant in the first pass and subsequent changes in deviation are much lesser in comparison to initial change. The Figures 4(a), 4(b) and 4(c) indicate the above-mentioned observations.
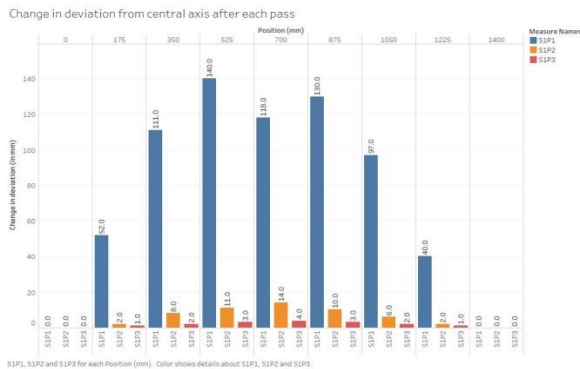

Figure 4(a)

Figure 4(a): Change in deviation from central axis after three passes for specimen-1
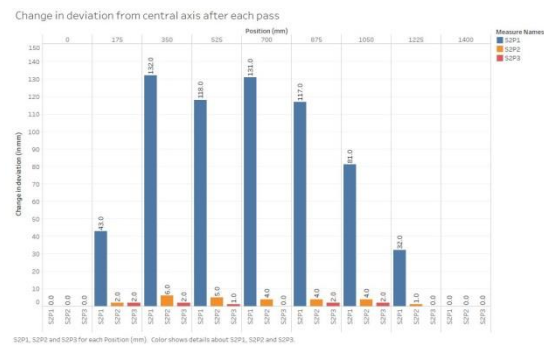

Figure 4(b)

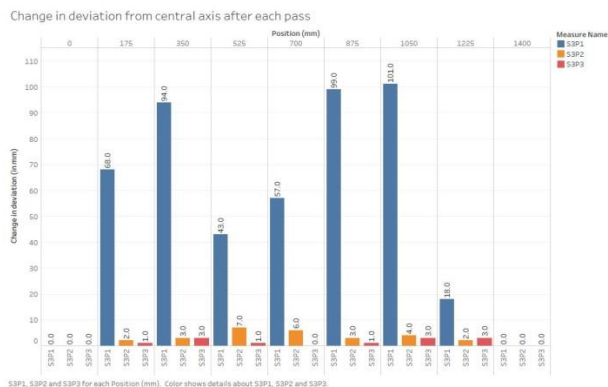**Figure 4(b):** Change in deviation from central axis after three passes for specimen-2

Figure 4(c)

Figure 4(c): Change in deviation from central axis after three passes for specimen-3

Although the data available is meagre in nature in the experimentation part, but it can be ascertained from above that with reduction of deflections, curvatures are actually reduced and degree of straightness is improved significantly with initial pass of reverse bending and also to some extent at subsequent passes. This is also in line with the very purpose of manufacturing of bar straightening machines. As each specimen enters the roller arrangement for reverse bending, bending moments thus faced by specimen causes improvement in reduction of curvature. However, due to insufficient experimental data, the frequency distribution followed by the reduction of deflection cannot be accurately ascertained.

It can be seen that moment and curvature have relationship in reverse kinematic loading for straightening of bar viz. bending moment M, initial and final curvatures κ including residual curvatures κ_r are therefore random variables in the event of straightening process which is actually a continuous one. As for continuous variables, probability density function in any arbitrary interval (a,b) or (λ_a, λ_b) of length segments of bar under reverse kinematic loading is applied.

However, values of interval (λ_a, λ_b) shall be close to the spacing between set of rolls producing bending moment and not exceeding the length between two rolls. Even if, rolls are somewhat evenly spaced, then too depending on throughput speed and residual stresses and residual curvatures, applied bending moments will be random in nature. This is more so, as bar under kinematic loading is also rotating about axis making an angle between loading plane and osculating plane affecting the process [6].

When a bent bar with initial residual curvature k_r is subjected to a loading moment in reverse direction (which may be assumed positive), an additional residual curvature k_r in the reverse direction causes it to become straightened, i.e. the resultant curvature tends to reduce to zero. However, in reality, it will be indeed probabilistic only, that meeting various parameters under consideration, i.e. speed of bar, angle between loading and osculating plane [Figure 4], applied load through cross rolls, the effect of actual reverse loading for curvature in opposite direction amounting same value in reverse direction to enable final residual curvature which shall tend to be zero at every point along the axis from beginning of process to end. However, there will indeed be various values of curvature at every point along the bar which will be random in nature.

Several variables are involved in the process of   bar straightening. These are viz. bending moment (M) arising due to reverse bending, angle (θ) between loading plane and osculating plane, throughput speed (v_x) of the bar in progress, section parameter of bar like radius of the bar (r), elastic and plastic moduli [E and E_p] of the bar, initial curvature (κ_r) and final curvature (κ_f).

An important addition to make here is that the zero probability of a random variable taking any selected value 'x' does not imply impossibility for the random variable taking on the value of x. Similarly, in the case of continuous assessment, zero probability does not correlate to logical impossibility. Additionally, owing to the practical limitations of our ability to measure and observe the entirety of the generated results, the study of this field is largely academic by nature and we ideally work within probabilities dealt in defined intervals and not for isolated points per se.

In the continuous case, probabilities associated with individual points are always zero, and therefore if we deal in the probability associated with the interval present between a and b, the inclusion of the endpoint becomes irrelevant. In the case of continuous intervals, the importance of specifying interval probabilities drops drastically.

As we know, P(X=x) =0 if X is continuous random variable. [9]

Symbolically,

$$P(a<X<b) = P(\lambda_a<X<\lambda_b) = P(\lambda_a<X \leq \lambda_b) = P(\lambda_a \leq X<\lambda_b) = P(\lambda_a \leq X \leq \lambda_b) \quad [18] \tag{9}$$
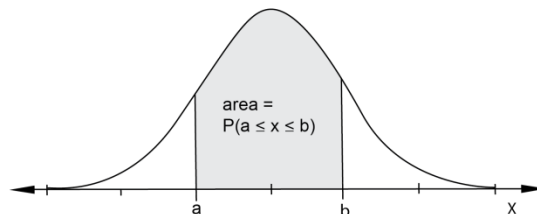

Figure 5: Probability as area under $f(x)$

Since, this is a continuous case of re-rolling, hence all the parameters figuring in the process will also be continuous random variables. The probability between any intervals, more specifically between the rolls of reverse bending shall be in consideration.

However, certain assumptions can be made which is quite logical and also to make our present case a bit simple. Since all rollers are running by standard commercial motors, revolution of these motors will be nearly constant or with insignificant variation which may be arising out of voltage fluctuation, if any. However, on stabilized power supply, the possibility of variation will actually be insignificant. Hence, throughput speed (v_x) of bars can be treated as constant excepting start and end points. Similarly, average radius of bar can also be treated as constant since dimensional variations will be insignificant and for all practical purpose considering variation is actually not much meaningful. Now, the angle between loading plane and osculating plane (θ) will be indeed varying. Roller positions are by and large fixed and rolling about their fixed axis. Variability aspect of θ can be considered suitably. Hence, at present, we can actually deal primarily with two main variables, i.e. change of bending moment ΔM and final curvature, κ_f.

Therefore, using equation (8), we can say that probability of occurrence of actual loading towards of change in bending moment under straightening process may be defined as

$$P(\lambda_a \leq \Delta M \leq \lambda_b) = P(\lambda_a \leq \kappa_f \leq \lambda_b) = \int_{\lambda_a}^{\lambda_b} f_\kappa(\kappa_f) \, d\kappa \tag{10}$$

where, $f_\kappa(\kappa_f)$ is the probability density function of final curvature due to change in bending moment on actual loading event during straightening process in the cross-roll arrangement.

Now as the value of μ_κ is available for a process, then by using the equation (10) it is possible to ascertain the value of final curvature κ_f at any interval region between a set of rolls causing reverse bending. The probability of any such value for κ_f can also be found out using equations (9) and (10).

Therefore, the probability or likelihood of falling values of final curvature κ_f within the range of application of reverse bending in the range of any set of rolls (λ_a, λ_b) or (a, b) will be as below:

$$P(\lambda_a \leq \kappa_f \leq \lambda_b) = \int_{\lambda_a}^{\lambda_b} f_k(\kappa_f) \, d\kappa \tag{11}$$

5. Discussion and Conclusion:
Probability density function can be evaluated when sufficient amount of experimental data is available. The probability distribution function of the data set will play significant role in ascertaining

the probability of occurrence of required straightness between set of rolls under reverse bending. Mean curvature and standard deviation of the bar can also be estimated accordingly.

The use of bar straighteners as a valuable preparatory process can be established by comparison between the conventional bar machining process and the use of bar straightening as a preliminary process before the machining stage. This can directly translate into improvements in multiple aspects such as productivity, energy requirements and monetary benefits.

Concept of statistical consideration has been applied to bar straightening process for estimation of final curvature of line segment of a given bar. The probability of occurrence of final curvature at a point within length segment can be expressed if probability density function is expressed between any two rollers in case of equidistant rollers. For any other arrangement of rollers, limit values of length segment of bar λ_a and λ_b have to be suitably changed and equation shall be changed accordingly. As the mean value of curvature and standard deviation of the bar after straightening process is within acceptable limits, same can be used for commercial purposes. The viability of using this for preparing raw material to produce various components increases as compared to current process.

Acknowledgements

References:

[1] Kemshall, G E 1974 Light Bar and Section Mill Technology, *Steel Times* **13** 243

[2] Das Talukder, N K, Singh A N 1991 *ASME J. Eng. Ind.* **113** 232.

[3] Meiwes Karl Christoph, Erdelen-Peppler, Marion and Brauer, Holger 2014 Canada. *Proc. of the 10th Int. Pipeline Conf. IPC2014*

[4] Wu B J, Chan L C, Lee T C, Ao L W 2000 *J. of Materials Proc. Technology*, **99** 202.

[5] Liscic B, Tensi H M and Luty W (Eds.), 1992 *New York, Springer-Verlag*

[6] Das Talukder N K, Singh A N 1991 *ASME J. of Engg. for Industry* **113** 224

[7] Davis E A, Rose B A, and Nadai A 1950 *Theory of Flow and Fracture of Solids*, 2nd *Ed. McGraw-Hill* **I** 20

[8] Nastran M and Kuzman K 2002 *J. of Materials Proc. Technology* **125-126** 711

[9] Tokunaga H 1961 *Bulletin of the J.S.M.E.* **4 15** 605

[10] Das Talukder N K, Johnson W, 1979 Varanasi. *Research Monograph No. 79121,* Mech. Engg. Dept., Banaras Hindu University

[11] Das Talukder, N K and Johnson W 1981 *Int. J. Mech. Sci.,* **23** 213

[12] Yu T X, Johnson W 1981 *Proc. 22 M.T.D.R. Con., Macmillan Press*

[13]   Tselikov A I, Smirnoff V V 1975 Oxford. *Straightening and Levelling Machines, Rolling Mills*, *Pergamon Press* **5** 254,

[14]   White J S, and Briggs G C, 1951 *Progress Report on Work on an Experimental Roller Leveler, Iron and Steel Research Association* Rep. No. MW/E/59/51,

[15] Ang A H S and Tang W H, *Probability Concepts in Engg. Planning and Design Vol-1, Basic Principle*

[16] Ang A H S and Tang W H, *Probability Concepts in Engg. Planning and Design Vol-2*, Decision, Risk and Reliability

[17]   Mitra Amitava Third Edition "Fundamentals of Quality Control and Improvement", Wiley India Pvt. Ltd. 613

[18] Chakrabarty J , 2014 *Theory of Plasticity*, Third Edition, Elsevier, Butterworth-Heinemann.

[19] Jain, RK, 2011 *Engineering Metrolog*, 20th Edition, Kahnna Publishers

[20] Johnson, Richard A, 2011 New Delhi, *Miller & Freund's Probability and Statistics for Engineers, PHI Learning Pvt. Ltd ,Eighth Edition* 121

[21] Casella George, Berger, Roger L, 2000 *Statistical Inference* 2nd Ed, Clenage Learning, 35

[22] Macura P , Petruska, J , 1996 *J. of Matl. Proc. Tech.,* **60**, 55

[23] Biju B, Dijin J S, Anujith C, Augustine, Arun, Mohammad, Anas P, 2016 *IRJET* **3** 379

[24] Groover Mikell P, 2010 John Wiley & Sons, Inc., *Fundamentals of Modern Manufacturing*, 472,